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Modelling merchantable volumes for uneven aged maritime pine (*Pinus pinaster* Aiton) stands established by natural regeneration in the central Portugal

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Abstract. Uneven aged maritime pine stands established by natural regeneration have a great expression in Portugal. These stands being overstocked, as opposed to those established from plantations, provide straight and cylindrical tree boles and logs with less knots that makes them very suitable for certain industrial purposes. Therefore, the aim of this study was to fit a set of equations to predict total volume and merchantable volumes to any merchantable limit for uneven aged maritime pine stands established by natural regeneration in the central inland region of Portugal. Data were collected in 30 circular sampling plots of 500 m^2 of area, on 1426 trees and 314 sample trees for volume assessment, corresponding to 2353 diameter/height measurements. A total height equation, a total volume equation, a volume ratio equation to any top height limit and a taper equation, over bark, were fitted. To select among the best models, several statistics were computed during model fitting and the independent validation procedure to evaluate model fitting, collinearity and prediction performances. A ranking index was used to support the final decision. The models selected were then fitted again using robust regression and weighted regression techniques, because studentized residuals distribution normality and homogeneity assumptions were not observed. This research showed that the models selected for these stands were not the same as those selected in previous studies for the species in this region, suggesting that these results may be due to the influence of stand density conditions on diameter and total height growth, and consequently, on stem form and volume. This set of equations will also be included as components in a single tree growth and yield model developed for these stands.
Keywords natural regeneration, uneven aged, total height equation, total volume equation, volume ratio equation, taper equation.

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Introduction

Models are absolutely essential for forest management planning and decision. Models make possible on determining values for individual logs and trees and for groups or stand trees offered as stumpage sales. They can also be useful in other common situations, for instances: for appraisal in forestry such as buying or selling timber or land, planning for future production and harvesting activities, obtaining financial compensation for damages, tax assets and income of the citizens, get a market value appraisal as condition to obtain a loan (Davis & Johnson 1987).

A variety of models are currently available to make empirical estimates of stand volume per acre, nevertheless, to make reliable predictions one must check a whether a model is appropriate for a particular management situation (acceptable accuracy, valid for the species and geographical area, and adapted to the available data)(Clutter et al. 1983, Davis & Johnson 1987, Vanclay 1994).

It is well known that stand's production efficiency is greatest at lowest stocking density that achieves full use of the site potential for timber production. However, in practice, others factors, such as, timber quality, price assumptions, harvesting costs, risk of windthrow and regeneration options, also need to be considered when determining desired stocking density (Skovsgaard & Vanclay 2008).

Spacing experiments have shown that denser stands of equal dominant height often have trees of smaller diameters than less denser stands. Thinning studies have indicated that stem diameter in the lower bole increases relatively faster in thinned stands than in comparables trees in unthinned stands. In general, isolated trees haveing more conical stem profile become more cylindrical as stand density increases. However, trees of the same diameter and total height will have different stem profiles, but may not necessarily have different stem volumes (Clutter et al. 1983). Thus, differences in stem form that result from stand density variation seldom have an economically significant impact on stem volume, but are of major importance on tree bole quality and merchantability and, hence, on its final industrial suitability (Loetsch et al. 1973, Husch et al. 1982, Avery & Burkhart 1983, Clutter et al. 1983, Philip 1994). For instance, there is field evidence, for maritime pine in Portugal, that the taper coefficient (*h*/*d*) of the open growth trees is smaller, when compared to the trees of the same dimension (either diameter or height) growing in stands. It can also be observed that the taper coefficient increases with both stand density and stocking (Alegria 2004). Furthermore, the ratio of the stump diameter to the diameter at breast height proved to be higher in even aged stands than in uneven aged stands showing differences in stem form (Carvalho 1992, Almeida 199).

Maritime pine (*Pinus pinaster* Aiton) is the most important species in Portugal, a country where forest land cover occupies 39% of the territory (3.5 x 10^6 ha) and belongs to non-industrial owners mostly (73.4%). The maritime pine stands represent 32% of Portuguese forest and 58% are located in the central region of the country where property average size is less than five hectares (Baptista & Santos 2005, DGRF 2006). Maritime pine usually grows in pure stands (65%) in structures ranging from even aged to multi aged stands, being 42% of them uneven aged (DGRF 2006).

A comparative study of maritime pine stands, in the central coastal region of Portugal, established from plantations and naturally regenerated, on tree bole merchantability proved that logs with fewer knots, straight and cylindrical bole suitable for pole production were found in naturally regenerated stands, as opposed to plantations stands, due to its overstocked growth conditions (Cabecinhas 2008).

Moreover, a study on economic efficiency of different silvicultural scenarios for maritime pine stands in Portugal showed that scenarios that led to a fully stocked stand situation resulted in the highest value for stand volume, mean annual increment and pulp wood volume (small size wood), and scenarios that resulted in under stocked stand situations were among the ones of higher round wood volume (large size wood). Good economic efficiency were found in scenarios combining high initial stand densities obtained from natural regeneration and cultural treatments that resulted in high stand stocking level (Alegria 2010a).

Studies carried out for maritime pine stands in the central inland region of Portugal (Carvalho 1992, Almeida 1994, Alegria 1993, Alegria 2004) showed that 85% of the maritime pine stands of the region are uneven aged due to the fact that these stands were installed by natural regeneration. In fact, these stands were regenerated over a period of several years, which can go up to 10-20 years, being observed tree age variations till to 39 years (Alegria 2004). It was also found that tree diameters standard deviations were small, being the diameter histogram a typically even aged one, even though tree age variation being greater than five years. This situation was reported by several authors that refer that in some cases, uneven aged stands may have a horizontal structure similar to even aged stands when they grow on poor sites (Husch et al. 1982, Clutter et al. 1983, Avery & Burkhart 1983, Davis & Johnson 1987). Based on several stocking indices, such as Wilson´s factor, stand density index and crown competition factor (Reineke 1933, Wilson 1946, Krajicek et al. 196, Clutter et al. 1983, Davis & Johnson 1987), it was also observed that about 71% of stands were overstocked, needing an appropriate thinning schedule, accordingly to stand stability (Alegria 2004).

On the other hand, a study on the mature maritime pine burned areas in 2003 showed that excellent level of regeneration were found after a post-fire period of five years (ranging from 300 to 50,000 trees per hectare with an average of 19,190 trees per hectare). This indicate that maritime pine stands reestablishment through natural regeneration still occurs after one or two fire cycles less than 20 years, as long as stored seeds in the soil and/or mature stands horizontal continuity exists (Martins 2007, Alegria 2010b).

Until recently, an equation for total volume over bark prediction developed by the National Forest Services (DGSFA 1969) was the only individual tree equation available for maritime pine stand volume prediction in the inland of Portugal in the Northern region of Tagus River. Several other models are now available: a system of equations for compatible prediction of total and merchantable volumes from Alegria (1993) and a total height equation from Almeida (1998), but none of them are specific for uneven aged maritime pine stands established by natural regeneration.

Because naturally regenerated, uneven aged, maritime pine stands are of strategic importance in this region and models for total and merchantable volumes prediction, to any defined merchantable specification to industrial use, are absolutely essential to forest management planning, the aim of this research is to fit a set of tree models, namely a total height model, a total volume model, a volume ratio model to any top height limit and a taper model, in order to support owners to explore the economic potential of these stands.

Materials and methods

Data collection

Data were collected in 30 circular plots of 500 m^2 of area, during September 1996 to April 1997, in uneven aged maritime pine stands provided by natural regeneration in the central inland region of Portugal. Growth variability conditions were ensured based on data collected in previous studies in 1992 and 1994.

The sample data were obtained from 1426 trees and 314 sample trees for volume evaluation, corresponding to 2353 tree bole diameter/height measurements (Table 1). Sample

Table 1 Data summary statistics

Stand variable (n = 30)		Minimum	Maximum	Mean	Standard deviation
N	(trees.ha^{-1})	460.0	1780.0	951.0	256.8
ddom	(cm)	11.6	36.0	28.9	5.2
hdom	(m)	5.0	19.2	15.5	2.9
Tree variable (n = 1426)					
d	(cm)	5.0	40.8	18.0	7.7
h	(m)	2.2	21.2	12.4	3.8
Sample tree variable (n = 314)					
d	(cm)	3.0	50.4	16.8	7.9
h	(m)	3.1	20.6	11.7	3.6
v	(m^3)	0.0024	1.2183	0.1770	0.2
Sample tree bole variable (n = 2353)					
d_h	(cm)	0.0	57.0	11.4	9.1
h_d	(m)	0.1	20.6	5.8	4.7
vm	(m^3)	0.0	1.2183	0.1355	0.3

trees were selected for measurement according to diameter at breast height class variability in the sample plot. Tree bole diameters over bark and heights were measured with Tele-relaskop (Loetsch et al. 1973). Individual tree total volume (v) and the accumulated merchantable volumes from the ground to each bole section (over bark) (*vm*) were obtained summing stump volume, bole sections volumes and tip volume. Stump volume was evaluated by the cylinder formula; each bole section volume was evaluated with Smalian's formula (Meyer 1953, Husch et al. 1982, Avery and Burkhart 1983); and tip volume with the cone formula.

Symbols

The symbols used in this study follow the criteria proposed by IUFRO (International Union of Forest Research Union) (van Soest et al. 1965). The variables used in the models are defined as follow:

b_i - regression coefficients estimated from the sample data, where i = 1, 2, 3, ...;
d - individual tree diameter, over bark, at breast height (1.30 m above ground) (cm);
ddom - dominant diameter (cm);
d_h - individual tree bole diameter, over bark (cm) as merchantable limit;
h - individual tree total height (m);
h_d - individual tree bole height (m) as merchantable limit;
hdom - dominant height (m);
N – number of trees per hectare (trees ha^{-1});
$p=h-h_d$;
r - individual tree volume ratio, vm/v, that multiplied by individual tree total volume, over bark, gives individual tree merchantable volume plus stump volume, over bark;
u_i - junction points of the sub models of the segmented polynomials models;
v - individual tree total volume, over bark, (m^3);
vm - individual tree volume, over bark, from the ground to some top diameter or top height limit (m^3);
$z = (h-h_d)/h$.

Candidate model selection

Individual tree height models predict individual tree height (h) as a function of individual tree diameter at breast height (d)(Loetsch et al. 1973, Husch et al. 1982, Avery & Burkhart 1983, Clutter et al. 1983). Tomé (1988) included a linear function of stand variables such as the number of trees per hectare (N) and the dominant diameter and height (*ddom* and *hdom*) to modify and improve these models. Individual tree height models were searched from Michailoff (1943), Stoffels & van Soest (1953), Prodan (1965), Harrison et al. (1986), Tomé (1988) and Almeida (1998). The study of the best linear combinations of stand variables to be included as a linear function, with intercept at $X = 0$, in the tree height models was made. Five modified candidate models were selected in a first trial for further analysis in this study (Table 2).

Individual tree volume models usually predict individual tree total volume (v) as a function of individual tree diameter at breast height (d) and individual tree total height (h)(Loetsch et al. 1973, Husch et al. 1982, Avery & Burkhart 1983, Clutter et al. 1983). Individual tree volume models were reviewed from Schumacher & Hall (1933), Stoate (1945), Naslung (1947), Spurr (1952), Meyer (1953), Takata (1958), Honer (1967), Ogaya (1968) and Burkhart (1977). In addition, 16 sub-models resulting from the linear combinations of two and three variables, with intercept at X=0, of the function $v = f(d, h, d^2, h^2, dh, d^2h, dh^2, d^2h^2)$ were also considered. Twenty candidate models were selected in a first trial for further analysis in this study (Table 3).

Individual tree volume ratio ($r = vm/v$) models estimate the ratio of merchantable volume (vm) to total (v) as a function of individual tree diameter at breast height (d), individual tree total height (h) and individual tree bole diameter (d_h) limit or individual tree bole height (h_d) limit (Burkhart 1977, Cao et al. 1980, van Deusen et al. 1981, Clutter et al. 1983). The application of individual tree volume ratio models needs an individual tree total volume model to convert volume ratio into merchantable volume to any bole diameter or height limit (Byrne & Reed 1986, Clutter et al. 1983). A review of individual tree volume ratio models was done

Table 2 Individual tree total height models

Model HE	Reference	Equation
1	Michailoff (1943) modified by Tomé (1988)	$h = hdom \exp^{\left(b_1 + b_2 hdom + b_3 \frac{N}{1000}\right)\left(\frac{1}{d} - \frac{1}{ddom}\right)}$
2	Stoffels and Van Soest (1953) modified by Tomé (1988)	$h = hdom \left(\frac{d}{ddom}\right)^{\left(b_1 + b_2 \frac{N}{1000}\right)}$
3	Prodan (1965) modified by Tomé (1988)	$h = hdom \left(1 + \left(b_1 + b_2 \frac{N}{1000}\right) hdom \left(\frac{1}{d} - \frac{1}{ddom}\right)\right)^{-1}$
4	Harrison et al. (1986) modified by Tomé (1988)	$h = hdom \left(1 + \left(b_1 + b_2 \frac{N}{1000}\right) \exp^{b_2 hdom}\right)\left(1 - \exp^{-b_3 \frac{d}{hdom}}\right)$
5	Harrison et al. (1986) modified	$h = hdom \left(1 + \left(b_1 + b_2 \frac{N}{1000} + b_3 ddom\right) \exp^{b_2 hdom}\right)\left(1 - \exp^{-b_3 \frac{d}{hdom}}\right)$

Table 3 Individual tree total volume models

Model VE	Reference	Equation
1	Schumacher and Hall (1933)	$v = b_1 d^{b_2} h^{b_3}$
2	Spurr (1952)	$v = b_1 \left(d^2 h\right)^{b_2}$
3	Honer (1967)	$v = \dfrac{d^2}{b_1 + \dfrac{b_2}{h}}$
4	Takata (1958)	$v = \frac{d^2 h}{b_1 + b_2 d}$
5	Burkhart (1977)	$v = b_0 + b_1 d^{b_2} h^{b_3}$
7	Spurr (1952)	$v = b_0 + b_1 d^2 h$
18		$v = b_0 + b_1 d^2 + b_2 dh^2$
20		$v = b_0 + b_1 dh + b_2 d^2 h$
22		$v = b_0 + b_1 d + b_2 d^2 h$
21		$v = b_0 + b_1 h + b_2 dh$
16		$v = b_0 + b_1 d + b_2 d^2 h^2$
17		$v = b_0 + b_1 d^2 h + b_2 d^2 h^2$
19		$v = b_0 + b_1 d^2 + b_2 d^2 h$
15		$v = b_0 + b_1 d^2 h + b_2 dh^2$
25		$v = b_0 + b_1 d^2 + b_2 h^2 + b_3 dh^2$
26		$v = b_0 + b_1 h^2 + b_2 dh + b_3 d^2 h^2$
27		$v = b_0 + b_1 h + b_2 d^2 + b_3 dh^2$
28		$v = b_0 + b_1 h^2 + b_2 dh + b_3 d^2 h$
29		$v = b_0 + b_1 h^2 + b_2 dh + b_3 dh^2$
30		$v = b_0 + b_1 h + b_2 dh + b_3 d^2 h^2$

from Honer (1967), Cao et al. (1980), Matney & Sullivan (1980), Reed & Green (1984) and Parresol et al. (1987). Four candidate models were selected in a first trial for further analysis in this study (Table 4).

Individual tree bole profile can be predicted with taper models which express individual tree bole diameter (d_h) as a function of the tree diameter at breast height (d), individual tree total height (h) and individual tree bole height (h_d) at the same measuring point (Kozak et al. 1969, Ormerod 1973, Demaerschalk 1973, Cao et al. 1980, Clutter et al. 1983). Individual tree taper models were reviewed from Kozak et al. (1969), Demaerschalk (1972), Bennett & Swindel (1972), Demaerschalk (1973), Ormerod (1973), Max & Burkhart (1976), Bennett et al. (1978), Clutter (1980), Biging (1984) and Parresol et al. (1987). Nine candidate models were selected in a first trial for further analysis

Table 4 Individual tree volume ratio models to any top height limit

Model VREh	Reference	Equation
1	Cao et al. (1980)	$r = 1 + \left[\frac{b_1 (h - h_d)^{b_2}}{h^{b_3}} \right]$
5	Parresol et al. (1987)	$r = \exp\left(b_1 z^{b_2}\right)$
6	Parresol et al. (1987)	$r = \exp\left[b_1\left(\frac{p^{b_2}}{h^{b_3}}\right)\right]$
7	Honer (1967)	$r = 1 + b_1\left(\frac{h_d}{h} - 1\right) + b_2\left(\frac{h_d^{\,2}}{h^2} - 1\right)$

in this study (Table 5).

The software SAS version 9.1 was used for model fitting and for the necessary computations along this study. The linear models were fitted by linear regression through the ordinary least squares method (OLS). The nonlinear models were fitted by nonlinear regression solved by the method of Gauss-Newton (Cody & Smith 1977, Der & Everitt 2000, SAS Institute Inc. 2004)

Criteria to evaluate model performance

To select the best individual tree total height, total volume, volume ratio to any top height limit and taper models for the species and region, candidate models were evaluated based on goodness of fit, collinearity and prediction performance (Draper & Smith 1981, Myers 1990).

The fitting performance of each model was evaluated with the statistics obtained in the SAS output which were: the squared multiple correlation coefficient (R^2), the adjusted squared multiple correlation coefficient (R^2adj) and the residual mean square (*RMS*) (Draper & Smith 1981, Myers 1990, SAS Institute Inc. 2004).

For the selection of the best individual tree total volume models among all models resulting from the linear combinations of two and three variables already referred to, the statistics R^2, R^2adj and Mallows Cp statistic were used (Draper & Smith 1981, Myers 1990, McQuerrie & Tsai 1998, Stauffer 2008). Thus, a first trial was done considering the best eight sub-models with two variables and with three variables for further analysis (Freund & Littell 2000, SAS Institute Inc. 2004).

An analysis was performed to detect high levels of model multicollinearity based on the computation of the variance inflation factors and the condition number (Myers 1990, Belsey 1991, Freund & Littell 2000, SAS Institute Inc. 2004). According to Myers (1990) a value greater than 10 was considered to indicate some collinearity in linear models. According to Belsey (1991) a condition number between five and 10 indicates that collinearity is not a major problem, in the range of 30 and 100 then

Table 5 Individual tree taper models

Model TE	Reference	Equation
2	Kozak et al. (1969)	$d_h = d\left[b_1\left(\frac{h_d}{h}-1\right)+b_2\left(\frac{h_d^2}{h^2}-1\right)\right]^{0.5}$
3	Kozak et al. (1969)	$d_h = d\left[b_1\left(1-\frac{2h_d}{h}+\frac{h_d^2}{h^2}\right)\right]^{0.5}$
4	Demaerschalk (1972)	$d_h = d\left(b_1 z^{b_2}\right)^{0.5}$
5	Demaerschalk (1972)	$d_h = b_1 d^{b_2}\left(h-h_d\right)^{b_3} h^{b_4}$
6	Demaerschalk (1973)	$d_h = d\left[b_1\left(\frac{1}{d^2 h}\right)\left(\frac{h-h_d}{h}\right)^{b_2}+b_3\left(\frac{h-h_d}{h}\right)^{b_4}\right]^{0.5}$
8	Ormerod (1973)	$d_h = d\left[\frac{h-h_d}{h-1.3}\right]^{b_1}$
10	Max and Burkhart (1976)	$d_h = d\left[b_1\left(\frac{h_d}{h}-1\right)+b_2\left(\frac{h_d^2}{h^2}-1\right)+b_3\left(u_1-\frac{h_d}{h}\right)^2 I_1\right]^{0.5}$ with, $I_1=1,\ \frac{h_d}{h}\le u_1$ $I_1=0,\ \frac{h_d}{h}>u_1$
12	Bennett et al. (1978)	$d_h = d\left(\frac{h-h_d}{h-1.3}\right)+b_1\left(\frac{\left(h-h_d\right)\left(h-1.3\right)}{h^2}\right)+b_2\left(\frac{d\left(h-h_d\right)\left(h-1.3\right)}{h^2}\right)+$ $+b_3\left(\frac{d^2\left(h-h_d\right)\left(h-1.3\right)}{h^2}\right)+b_4\left(\frac{\left(h-h_d\right)\left(h-1.3\right)\left(2h-h_d-1.3\right)}{h^3}\right)$
16	Bennett and Swindel (972)	$d_h = b_1 d\left(\frac{h-h_d}{h-1.3}\right)+b_2\left(h-h_d\right)\left(h_d-1.3\right)+b_3 h\left(h-h_d\right)\left(h_d-1.3\right)+$ $+b_4\left(h-h_d\right)\left(h_d-1.3\right)\left(h+h_d+1.3\right)$

problems associated with collinearity exist and in the range of 1000 and 3000 then collinearity problems are severe. In this study, a condition number greater than 100 was considered as criteria to exclude a model from further analysis. Prediction performance of each model was evaluated through the analysis of the Press residuals (Myers 1990). Based on the Press residuals the following statistics were computed: the average of the Press residuals (*PRESS*), the average of the absolute values of the Press residuals (*APRESS*) and the average of the sum of squares of the Press residuals (*SPRESS*).

An overall ranking index (*RI Overall*), to support best model selection for total height, total volume models, volume ratio models to any top height limit and taper models, was obtained as the average of the computed fitting and prediction ranking indexes (*RI Fitting* and *RI Prediction)*). Model fitting ranking index was obtained as the average of the statistics ranking indices: $RI(R^2)$, $RI(R^2adj)$ and $RI(RMS)$. Model prediction ranking index was obtained as the average of the statistics ranking indices: *RI*(*PRESS mean*), *RI*(*APRESS mean*) and *RI*(*SPRESS mean*).

Statistics ranking indices were defined in the interval of [0, 1], where the model with the best value of the statistics had an index value of one and the model with the worst value had an index value of zero. The ranking indices for $RI(R^2)$ and $RI(R^2adj)$ were computed as exemplified below for $RI(R^2)$:

$$R\,I(R^2) = \frac{I_2}{MAX(I_2)} \tag{1}$$

with $I_2 = I_1 - MIN(I_1)$ and $I_1 = 1 - \frac{MAX(R^2) - R^2}{MAX(R^2)}$

The ranking indices *RI*(*RMS*), *RI*(*PRESS mean*), *RI*(*APRESS mean*) and *RI*(*SPRESS mean*) were computed as exemplified below for *RI*(*RMS*):

$$R\,I(RMS) = \frac{I_2}{MAX(I_2)} \tag{2}$$

with $I_2 = I_1 - MIN(I_1)$ and

$$I_1 = 1 - \frac{MIN(RMS) - RMS}{MIN(RMS)} .$$

The studentized residuals (*STR*) were computed to analyse the errors distributions of the models for normality and homogeneity (Myers 1990, Freund & Littell 2000, SAS Institute Inc. 2004). The assumption of normality of the studentized residuals was tested using the normality tests of Kolmogorov-Smirnov and of Anderson-Darling ($n > 50$) and by graphical evaluation (Mason et al. 1989). If the tests *P*-value was lower than the $\propto = 0.05$ level of significance then the hypothesis that these residuals have a normal distribution was rejected. The analysis of the studentized residuals probability plots (Devore & Peck 1997, Der & Everitt 2000, Freund & Littell 2000, SAS Institute Inc. 2004) was done to test both non-normality and non-homogeneity of the variance of errors distribution (Myers 1990). Simultaneously, a study of possible outliers has been carried out through the analysis of studentized residuals versus predicted values plot (Myers 1990).

If deviation of regression model assumptions was found for the best models selected, then the models were fitted again through robust regression techniques, to attempt to overcome the problems of non-normal residuals distribution and correlated explanatory variables (Vanclay 1994). An iteratively reweighed least squares method (IRLS) with the Huber's influence function for parameter estimation was used to attenuate the influence of data points containing large errors in m odel fit (Myers 1990, SAS Institute Inc. 2004). This technique also provides qualitatively superior parameter estimation than the OLS method (Myers 1990).

In the presence of normality, independence, and heterogeneous variance, the weighted regression (WLS) with a variance estimating function w_i that is inversely related to the variance of the errors at each data point was used as well (Draper & Smith 1981, Myers 1990, SAS Institute Inc. 2004, Vanclay 1994, Freund & Littell 2000).

To analyse the predictive performance of the models selected and the models in use in the region, several validation statistics were obtained through residuals analysis, based on the prediction residuals (true value minus predicted value), namely: the model bias (as the mean of prediction residuals), the model accuracy (as the absolute mean of prediction residuals) and the model precision (as the prediction residuals coefficient of determination) (Huang et al. 2003).

Results

Based on the statistics determined for the tested models (Table 6), model HE1, the model from Michailoff (1943) modified by Tomé (1988), an exponential model that had been modified by including the stand variables *hdom*, *ddom* and *N*, which has a logical biological behaviour (with asymptote), was selected. This model had the best overall ranking index. Nevertheless, a moderate level of collinearity was observed (condition number of 22.3).

After verifying regression models assumptions through the analysis of studentized residuals it was observed that the studentized residuals normality assumption was rejected for all the studied models.

To overcome this situation, the model selected (HE1) was fitted again through the IRLS method. The studentized residuals plot, the studentized residuals normal probability plot after fitting the model through the OLS and the studentized residuals normal probability plot after fitting through the IRLS method are presented in figure 1.

Based on the statistics determined for the tested models (Table 7), model VE25, a linear model resulted from the linear combination of the variables d^2, h^2 and dh^2, with intercept at $X = 0$, was selected. This model had the best overall ranking index, with the second best ranking index for fitting and the best ranking

Table 6 Statistics for individual tree total height models

Model	R^2adj	RMS	PRESS mean	APRESS mean	Condition number	RI Fitting	RI Prediction	RI Overall
HE1	**0.8740**	**1.83650**	0.01030	**1.04593**	22.3	**1.000**	**0.989**	**0.994**
HE2	0.8423	2.29870	0.20261	1.19962	7.5	0.000	0.000	0.000
HE3	0.8680	1.92350	0.08976	1.08441	7.5	0.811	0.734	0.772
HE4	0.8631	1.99450	-0.00115	1.11272	17.6	0.660	0.720	0.690
HE5	0.8678	1.92590	**-0.00399**	1.07972	57.2	0.810	0.844	0.827

Note: Best values are highlighted in bold

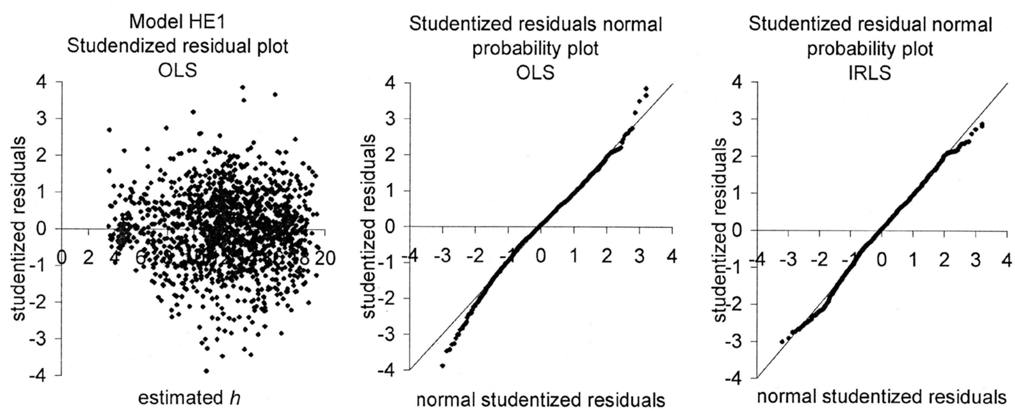

Figure 1 Studentized residuals for the individual tree height model selected

Table 7 Statistics for individual tree total volume models

Model	R^2adj	*RMS*	*PRESS* mean	*APRESS* mean	Condition number	*RI* Fitting	*RI* Prediction	*RI* Overall
VE1	0.9214	0.00265	-0.00287	0.02512	48.0	0.842	0.622	0.732
VE2	0.9217	0.00264	-0.00276	0.02488	27.0	0.855	0.679	0.767
VE3	0.9157	0.00284	0.00189	0.02476	12.5	0.312	0.512	0.412
VE4	**0.9231**	**0.00259**	-0.00120	**0.02379**	7.3	**0.988**	0.895	0.942
VE5	0.9219	0.00263	-0.00038	0.02566	62.5	0.913	0.874	0.893
VE7	0.9122	0.00296	-0.00021	0.02652	2.3	0.000	0.527	0.264
VE18	0.9209	0.00267	-0.00005	0.02667	9.1	0.802	0.914	0.858
VE20	0.9194	0.00271	-0.00041	0.02560	13.4	0.665	0.778	0.722
VE22	0.9193	0.00272	-0.00025	0.02636	12.7	0.657	0.794	0.726
VE21	0.9182	0.00276	0.00007	0.02944	16.0	0.556	0.813	0.684
VE16	0.9175	0.00278	-0.00009	0.02750	9.8	0.496	0.780	0.638
VE17	0.9157	0.00284	-0.00011	0.02545	20.4	0.332	0.659	0.496
VE19	0.9151	0.00286	-0.00017	0.02579	18.2	0.273	0.637	0.455
VE15	0.9148	0.00287	-0.00060	0.26061	11.9	0.251	0.258	0.255
VE25	0.9228	0.00260	-0.00011	0.02446	23.4	0.985	**0.983**	**0.984**
VE26	0.9225	0.00261	-0.00011	0.02570	18.7	0.962	0.972	0.967
VE27	0.9224	0.00262	-0.00011	0.02486	20.2	0.948	0.963	0.956
VE28	0.9223	0.00262	-0.00032	0.02590	29.0	0.944	0.909	0.926
VE29	0.9220	0.00263	0.00004	0.02616	25.9	0.916	0.963	0.940
VE30	0.9214	0.00265	**0.00000**	0.02602	25.8	0.862	0.949	0.905

Note: Best values are highlighted in bold

index for prediction, with a condition number of 23.4 showing moderate level of collinearity.

The analysis of studentized residuals was also accomplished for the tree total volume models. Both, the studentized residuals normality and homogeneity assumptions were rejected for all the studied models.

Therefore, the model selected (VE25) was fitted again through the WLS and IRLS methods. In figure 2, the studentized residuals plot, the studentized residuals normal probability plot after fitting the model through the OLS and the studentized residuals normal probability plot after fitting through both WLS and IRLS methods are presented.

From the analysis of table 8, model VREh1, a model from Burkhart (1977) and modified by Cao et al. (1980), was selected. This model has the best overall ranking index and the second best ranking indices for fitting and prediction. Once again, a moderate level of collinearity was observed (condition number of 49.4).

The analysis of studentized residuals showed that studentized residuals normality assumption was rejected for all the studied models.

The model selected (VREh1) was fitted again through the IRLS method. Figure 3 shows the studentized residuals plot, the studentized residuals normal probability plots after fitting the model through the OLS and the studentized residuals normal probability plot after fitting through the IRLS method.

The taper model selected was the model TE10, the model from Max & Burkhart (1976), a segmented polynomial function of two submodels with one joint point, which had the best overall ranking index, the best ranking indices for fitting and prediction (Table 9) and a moderate level of collinearity (condition number of 17.6).

The analysis of studentized residuals showed that studentized residuals normality assumption was as well rejected for all the studied models.

The model selected (TE10) was fitted again through the IRLS method. The studentized residuals plot, the studentized residuals normal

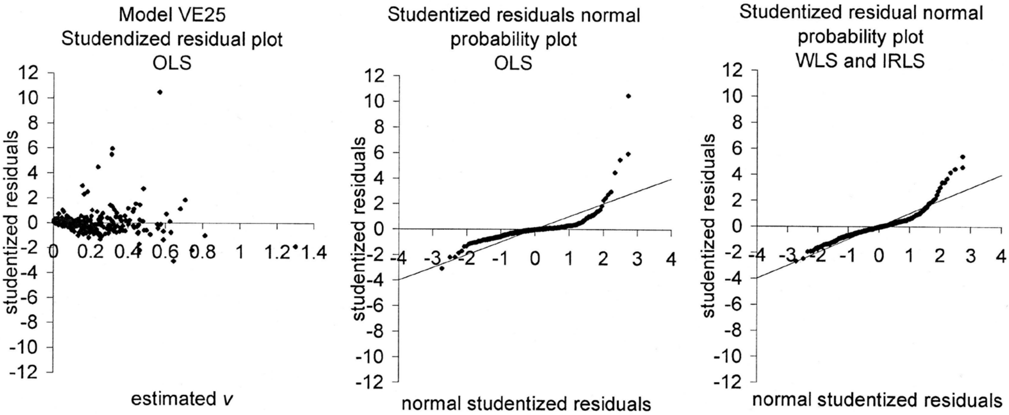

Figure 2 Studentized residuals for the individual tree volume model selected

Table 8 Statistics for individual tree volume ratio models to any height limit

Model	R^2adj	RMS	PRESS mean	APRESS mean	Condition number	RI Fitting	RI Prediction	RI Overall
VREh1	**0.9874**	**0.00143**	**-0.00303**	**0.02556**	49.4	**1.000**	**1.000**	**1.000**
VREh5	0.9668	0.00377	-0.01570	0.04760	2.8	0.000	0.000	0.000
VREh6	0.9671	0.00374	-0.01552	0.04756	41.4	0.013	0.010	0.011
VREh6	0.9805	0.00222	-0.00487	0.03682	13.1	0.663	0.668	0.666

Note: best values are highlighted in bold

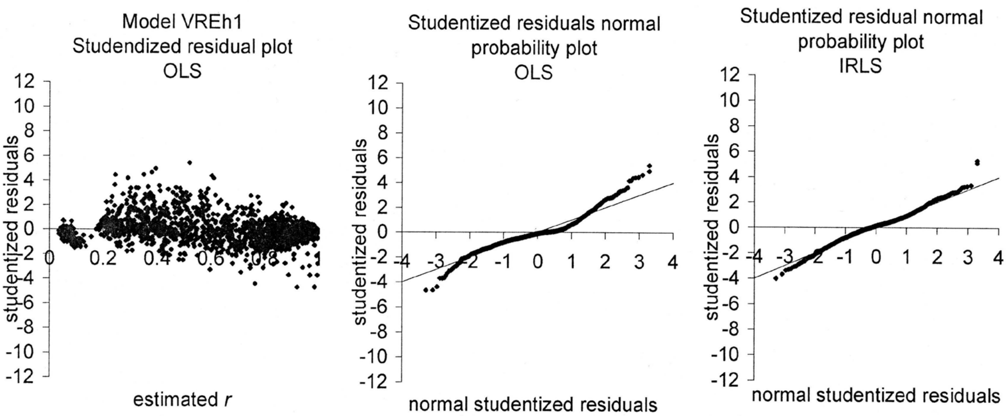

Figure 3 Studentized residuals for the individual tree volume ratio model to any top height limit selected

probability plots after fitting the model through the OLS and the studentized residuals normal probability plot after fitting through the IRLS method can be seen in figure 4.

Finally, the set of fitted equations, with their sample size, their parameter estimates obtained through the IRLS method or both the IRLS and WLS methods, and their fitting and prediction

Table 9 Statistics for individual tree taper models

Model	R^2adj	RMS	PRESS mean	APRESS mean	Condition number	RI Fitting	RI Prediction	RI Overall
TE2	0.9398	4.31020	0.18990	1.42278	9.8	0.092	0.255	0.173
TE3	0.9386	4.39830	0.30717	1.46971	1.0	0.016	0.100	0.058
TE4	0.9391	4.36420	0.23604	1.43906	2.1	0.046	0.187	0.116
TE5	0.9439	4.01710	**0.03683**	1.40834	48.5	0.344	0.483	0.414
TE6	0.9522	3.42310	0.07075	**1.23117**	8.9	0.851	0.905	0.878
TE8	0.9384	4.41660	0.46702	1.31649	1.0	0.000	0.161	0.080
TE10	**0.9547**	**3.24910**	0.15870	1.24706	17.6	**1.000**	**0.913**	**0.956**
TE12	0.9431	4.07560	0.13083	1.45076	21.8	0.293	0.360	0.326
TE16	0.9435	4.05206	0.09399	1.41375	26.7	0.315	0.429	0.372

Note: best values are highlighted in bold

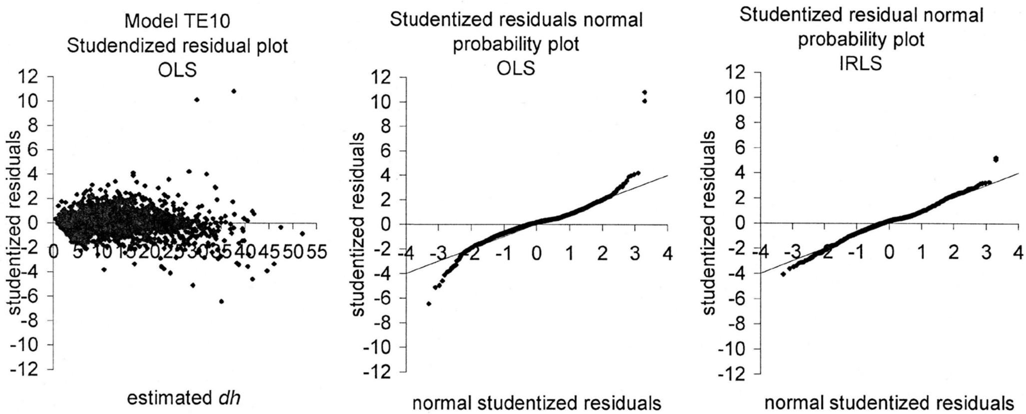

Figure 4 Studentized residuals for the individual tree taper model selected

statistics are summarized in Table 10.

Discussion

A set of models for individual tree merchantable volume predictions were fitted for the uneven aged maritime pine stands established by natural regeneration. The models selected were: the total height model from Michailoff (1943) modified by Tomé (1988) with four parameters *d*, *hdom*, *ddom* and *N*; the total volume model a linear equation, with intercept at $X = 0$, of three parameters d^2, h^2 and dh^2; the volume ratio model to any top height limit from Cao et al. (1980) with two parameters *h* and *hd*; and the taper model from Max & Burkhart (1976), a segmented polynomial function of two sub-models with one joint point, that uses relative heights and has three parameters *d*, *h* and *dh*.

The analysis of studentized residuals for the models selected showed that residuals normality assumption was not met; for the total volume model the residuals homogeneity assumption was also not met. Based on the analysis of models bias, accuracy and precision it was proved that the models selected, after being fitted by again through the IRLS method or both the WLS and IRLS methods, kept on performing better than the candidate models unselected.

The models selected were not the same as those selected in previous studies for the spe-

Table 10 Selected models

Models and statistics

HE - Individual tree height equation

$$h = hdom \exp^{\left(-4.2759-0.4152hdom+3.0459\frac{N}{1000}\right)\left(\frac{1}{d}-\frac{1}{ddom}\right)} \qquad 1$$

$R^2 = 0.8742$; $R^2adj = 0.8740$; $RMS = 1.83650$; $Mean\ PRESS = 0.01030$;

$Mean\ APRESS = 1.04593$; $n = 1426$; parameters obtained through the IRLS method.

VE - Individual tree total volume equation

$$v = 0.00403 + 0.00026001d^2 - 0.00034091h^2 + 0.00004263hd^2 \qquad 4$$

$R^2 = 0.9235$; $R^2adj = 0.9228$; $RMS = 0.00260$; $Mean\ PRESS = -0.00011$;

$Mean\ APRESS = 0.02446$; $n = 314$; parameters obtained through the IRLS and WLS methods.

VREh - Individual tree volume ratio equation to any top height limit

$$r = 1 + \left[\frac{-0.9201(h-h_d)^{2.8138}}{h^{2.7901}}\right] \qquad 1$$

$R^2 = 0.9874$; $R^2adj = 0.9874$; $RMS = 0.00143$; $Mean\ PRESS = -0.00303$;

$Mean\ APRESS = 0.02556$; $n = 2353$; parameters obtained through the IRLS method.

TE - Individual tree taper equation

$$d_h = d\left[-2.0845\left(\frac{h_d}{h}-1\right)+0.936\left(\frac{h_d^2}{h^2}-1\right)+51.9435\left(0.1005-\frac{h_d}{h}\right)^2 I_1\right]^{0.5}$$

$$\text{with, } I_1 = 1,\ \frac{h_d}{h} \le u_1$$

$$I_1 = 0,\ \frac{h_d}{h} > u_1 \qquad 10$$

$R^2 = 0.9547$; $R^2adj = 0.9547$; $RMS = 3.24910$; $Mean\ PRESS = 0.15870$;

$Mean\ APRESS = 1.24706$; $n = 2353$; parameters obtained through the IRLS method.

cies in the study area. Namely, Almeida (1998) selected the model from Prodan (1965) modified by Tomé (1988), a hyperbolic equation modified to include the stand variables *hdom*, *ddom* and *N*, with logical biological behaviour; Alegria (1993) selected the total volume model from Spurr (1952), a linear model of the variable d^2h, with intercept at $X = 0$; and the taper model from Demaerschalk (1973), a polynomial function of three parameters *d*, *h* and *dh*, that uses both relative heights and diameters. Only the volume ratio model to any top height limit from Cao et al. (1980) was the same in both studies.

The differences on models selection for total height, total volume and stem profile predictions may be due to the influence of stand density conditions on diameter and total height growth, and consequently, on stem form and volume. The selection of the segmented taper model, with one joint point, from Max & Burkhart (1976), a more versatile and robust model, reinforces this argument.

In fact, Byrne & Reed (1986) argued that segmented polynomial models provide a better tree stem fitting, especially in bole's base, and that they can describe the different shape's sections of tree bole in a more suitable way.

When comparing the prediction performance of the models selected to the models in use for the species in the region before, a decrease on bias and an improvement on both accuracy and precision was observed (Figure 5).

At the same time, it is interesting to stress that the volume ratio equation to any top height limit from Cao et al. (1980) and the taper equation from Demaerschalk (1973) has been selected in a study for *Eucalyptus globulus* Labill. plantations in Portugal (Tomé 1991); the taper equation from Max & Burkhart (1976) was also selected in several studies, for instance: for *Eucalyptus globulus* Labill in Portugal (Tomé 1991), for natural stands and plantations of *Pinus taeda* L. in USA (Cao et al. 1980), for plantations of *Pinus taeda* L. in Southern Brazil (Figueiredo-Filho et al. 1996) and for mixed stands of conifers in North California, USA (Biging 1984). In fact, Cao et al. (1980) suggested that those models would possibly perform well for other coniferous trees.

In short, some reviewed models perform well in a variety of situations ranging from specie to region and growth conditions. This may be considered of great interest in including them as candidate models in future modelling studies of this kind.

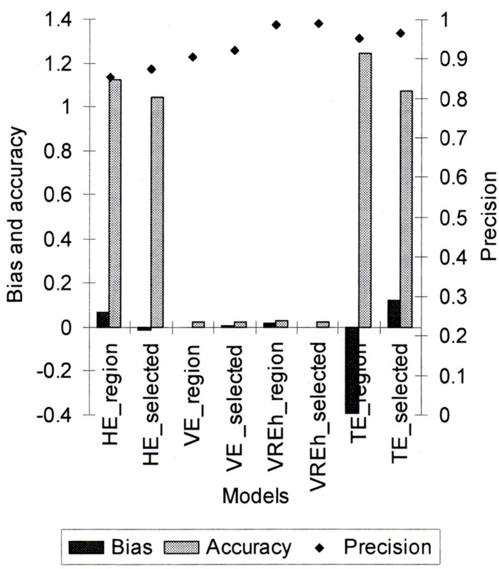

Figure 5 Bias, accuracy and precision for the mod els selected and the models in use for the species in the region

Conclusion

A set of equations - a total height equation and a total volume equation, a volume ratio equation to any top height limit and a taper equation, over bark - for the uneven aged maritime pine stands established by natural regeneration in the central inland of Portugal were fitted. These equations enable the prediction of total volume and merchantable volume, over bark, to any merchantable limit.

Individual tree bole diameters, over bark, can be simulated using the taper equation to search minimum log height for the dimension class of timber industrial use. Subsequently, using the ratio volume equation - to the searched height merchantable limits found earlier - in conjunction with the total volume equation, merchantable volume, over bark, will be simulated.

These models were proved to have a better fitting and prediction performances than the existing models and can be safely used in a growth and yield model for the uneven aged stands of the species in the study area.

They will also be of major importance on developing the Forest Management Plans required by law - for all forest private areas with more than 25 hectares and/or for the Intervention Forest Zones – in order to promote sustainable forest management and minimize forest fire risk (DR 2009, AFN 2009).

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