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Taper functions and merchantable timber for temperate forests of northern Mexico

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Abstract. Taper functions are required in modern forest management in estimation of the end forest products, to be classified for their life time in the environment. Based on a sample of 1640 trees of 10 species measured in volume, biomass and taper project on Mexico's northern temperate, mixed, uneven-aged coniferous forests, 12 stem profile taper functions were fitted in order to select the equation that provides better diameter estimates at commercial tree height. Although several equations fitted better specific tree species, the Newnham (1990) equation consistently yielded better diameter estimates at any length of the stem for all studied species. The confidence intervals on the Newnham (1990) equation parameters showed that each species has an unique stem profile and, therefore, single parameter equations are reported. Because of lack of analytical integration, the recommended taper equation (when numerically integrated) provided compatible, unbiased total bole volume when contrasted to conventional timber volume assessments. Data for 637 circular, 1/10 ha, plots from temperate forests of Central Durango, Mexico estimated a mean of 135 m^3 ha^{-1}, of which 18, 59, 30, and 17 m^3 ha^{-1} could be classified as poles, sawn wood, plywood and secondary forest products, respectively. This information can be used for the planning of the forest industry to optimize forest products derived from timber harvesting, as well as for estimating other environmental components.
Keywords taper functions, Newnham (1990), variable exponent function.

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Introduction

In the past, the native coniferous forests provided society with a broad diversity of goods. Demographic growth and the increasing demand for food led to the deforestation of vast forest areas. Therefore, modern conventional and sustainable forest management requires an increment on the efficiency of forest products derived from timber harvesting of a decreasing area of native forests. Taper (stem profile) functions provide an alternate approach to volume estimation for the different merchantable portions of a tree bole (Kozak et al. 1969, Clutter et al. 1983, Bailey 1994). Taper equations predict diameter at any point on the bole height or length. The taper function integration for a bole portion estimates the volume contained in that stem portion, therefore, allowing a flexible size classification for forest products.

At present, there are other pressing environmental reasons for the classification of forest products derived from timber harvesting: the determination of the accumulated biomass for each fraction of the tree, as the size and quality of the portion will influence its end use, its half-life in the environment, and therefore its time to return carbon to the atmosphere (Montero et al. 2002). Also, several bird species require trees within certain dimensional range to perform part of their lifecycles, e.g., tall and high-diameter trees for prey and carpenter birds.

A wide range of taper functions exists in the literature, which can be broadly classified as: (i) segmented polynomial models (Cao et al. 1980, Max-Burkhart 1976, Parresol & Thomas 1996), (ii) simple polynomial models (Kozak et al. 1969, Goulding & Murray 1976), (iii) power models (Clutter 1980, Damaerschalk 1972, Amidon 1984, Bigging 1984, Newnham 1990), (iv) variable exponent models (Newnham 1988), (v) geometry oriented models (Parresol & Thomas 1996, Fang & Bailey 1999, Zhang et al. 2002). Shifts in stem tapering between trees of the same stand and between trees of different stands, in addition to the different mathematical perceptions on the mode of addressing the description of the stem profile, have led to the abundance of taper functions (Gregoire & Schabenberger 1996, Tassisa & Burkhart 1998, Eerikäinen 2001). Newnham (1988) stated the two reasons for continuous study in this area: (i) no single theory has been developed that adequately explains the variation in stem form for - different trees, (ii) as a method of estimating volume, a single taper equation can estimate both total and merchantable tree stem volume. If tree form can be accurately described, then volume for any merchantability limit can be more precisely predicted.

Native pine-oak forests of Northern Mexico contain more than two thirds of standing timber volume and provides with more than 70% of the timber officially harvested in Mexico (SEMARNAT 2012). Pine species followed by oak trees, are the main genus harvested and sawn wood is the main end product derived from timber. There is little information on how to objectively estimate the sawn wood and other end forest products derived from timber for all the temperate forests of northern Mexico. Several stem profile functions have been reported for specific pine species and for specific sites of northern Mexico (Aguirre-Bravo 1987, Návar et al. 1997, Corral-Rivas et al. 1999, Corral-Rivas et al. 2007). However, these reports miss several commercial timber species (*P. menziesii*, *P. arizonica*, *P. ayacahuite*, *P. hartwegii*, and *Quercus* sp.) and they provide accurate stem profile assessments for specific sites of temperate forests.

Therefore, the aim of this work was to fit taper equations already available in the scientific literature for merchantable tree volume estimation, which allows flexible end-product classification schemes for pine species and oak trees harvested all over northern Mexico.

Materials and methods

Study site. This research was conducted on the ejidos, comunal forests, of southern Chihuahua (e.g., Papajichi, Guachochi and Caborachi) and Durango (e.g., El Tarahumar, El Tule, La Soledad, El Negro, Altares, Valle de Topia, San Miguel, Salto de Camellones, Milpas, Tambores, San Manuel de Villa Corona, San Luis de Villa Corona, San Bartolo and Santiago Teneraca) and in the State of Nuevo Leon, Mexico. The ejidos of Durango and Chihuahua are located in the temperate pine-oak forests of the Sierra Madre Occidental mountain range, where the mean annual temperature and rainfall are about 12°C and 900 mm, respectively. Soils in upland sites are characterized by Litosols and Regosols.

Species. The research aimed at measuring and predicting taper, volume and biomass components of a large number of temperate trees. For this 10 tree species were chosen, because they spanned a broad range of phenological and physiological traits and were species known to distribute well in temperate pine-oak forests of the Sierra Madre Occidental and Oriental mountain ranges of northern Mexico (SMW and SMO, respectively hereafter). All of them are native to the studied forests, and all are species of economic importance, since are harvested for industrial timber and other secondary forest products such as toothpicks, lollipop sticks, broom and mop handles, scaffolds, and wood boxes.

Pine-oak forests of northern Mexico. The SMW boast extensive areas of pine-oak forests that contain more than two thirds of the standing timber in Mexico. The western slopes of the Sierra Madre Occidental holds a climate that is generally somewhat wetter than the Sierra Madre Oriental mountain range, with presumably milder winter temperatures, and resulting in a more diverse flora, with more tropical elements, including Apache pine (*Pinus engelmannii*), Durango pine (*P. durangensis*), egg-cone pine (*P. oocarpa*), pino chino (*P. herrerae*), and Mexican tropical-montane oaks. Pine-oak forest continuous with oak woodland at lower elevations. At higher elevations, within the pine-oak zones, the pines become increasingly conspicuous and the tree density increases. In Chihuahua, pine forest is characteristically dominated by one species usually Arizona pine (*Pinus ponderosa* var. *arizonica*) with scattered individuals or small groups of oaks, especially Gambel oak (*Q. gambelii*) and net-leaf oak (*Q. rugosa*). In Durango, *P. cooperi* Ornelasi dominates the overstory and *Q. sideroxyla* the understory of most pine forests of the higher elevations.

The Sierra Madre Oriental mountain range is home of the dominant pine species *Pinus nelsonii*, *P. cembroides*, *P. pseudostrobus* and *P. arizonica*, together wich oaks *Quercus castanea* and *Q. affinis*. The associations between *Pinus* and *Quercus* with other species vary, depending on altitude and humidity of the areas they inhabit along the Sierra Madre Oriental.

Samples. Due to the small diameter variance (INF 2004-2010), the experimental design consisted on selection of approximately 50 trees per ejido that were harvested during commercial operations. Trees were sampled proportionally to the diameter distribution of the forest inventory (1997) for the State of Durango, Mexico. Trees were felled and diameters measured at the base, at 0.50, at 1 m, at 1.3, at 2.0 and then every meter thereafter on felled trees, where top height was also measured. A total of 1640 trees were harvested for stem profile measurements (Table 1): 1542 *Pinus spp* (specifically 214 trees of *P. psedostrobus* Lindl., 128 trees of *P. hartwegii* Lindl., 64 trees of *P. cooperi* C.E. Blanco, 45 trees of *P. ayacahuite* Ehrenberg ex Schlechtendahl 1838, 423 trees of *P. durangensis* Martinez, 84 trees of *P. leiophylla* Schiede ex Schlectendal et Chamisso 1831, 397 trees of *P. teocote* Schiede ex Schlectendal et Chamisso, 126 trees of *P. arizonica* Engelmann, and 61 trees of other coniferous species) and 98

Table 1 Summary statistics for sampled trees used to fit taper functions in temperate, uneven-aged, mixed pine-oak forests

	Top height (m)				Diameter at breast height (cm)				
	Mean	Min	Max	SD	Mean	Min	Max	SD	N
P. pseudostrobus	16.8	7.8	33.0	4.6	27.8	11.0	53.0	11.3	214
P. hartwegii	12.5	3.7	18.5	2.5	36.1	17.9	59.7	9.4	128
P. cooperi	19.0	5.8	29.7	5.3	32.0	8.2	57.4	11.4	64
P. ayacahuite	11.5	4.2	19.8	4.6	15.5	5.7	30.3	6.6	45
Q. spp	14.8	7.9	25.3	3.8	22.8	8.7	44.8	7.7	61
P. durangensis	15.3	5.3	32.0	4.4	23.4	6.2	62.0	10.3	423
P. leiophylla	13.9	6.2	27.0	4.9	23.8	9.6	56.0	9.8	84
P. teocote	16.6	6.5	33.0	4.1	26.2	7.3	53.0	9.8	397
P. arizonica	14.7	6.4	27.5	3.6	24.8	9.9	45.0	7.3	126
Quercus spp.	12.4	4.0	23.9	3.5	26.3	7.3	62.5	9.9	98

Note. Abbreviations: Min - minimum, Max - maximum, *SD* - Standard Deviation, *N* - sample size.

Quercus sp. Pseudotsuga menziesii Mirb. Franco trees were pooled together with pine species that have a small number of individuals to develop one equation for the group of trees called other coniferous spp even though mixing genus is a grave violation of the assumptions used to develop taper equations.

Taper models tested. A total of 12 mathematical functions that predict the stem profile[1]: Cao et al. (1980), Parresol & Thomas (1996), Max & Burkhart (1976), Kozak et al. (1969), Kozak et al. (1969), Goulding & Murray (1976), Damaerschalk (1972), Newnham (1990), Newnham (1988), Amidon (1984) and Bigging (1984).

The segmented taper functions of Cao et al. (1980), Parresol & Thomas (1996) and Max & Burkhart (1976) are equations 1-3. The polynomial functions of Kozak et al. (1969), Kozak et al. (1969), Goulding & Murray (1976) are equations 4-6. The stem profile power models of Clutter (1980), Damaerschalk (1972), Newnham (1990) are equations 7-9. The equation of the variable exponent of Newnham (1988) is the model from equation 10. The taper functions of Amidon (1984) and Bigging (1984) are displayed in equations 11-12.

$$\frac{d^2 \cdot K \cdot H}{V_m} - 2Z = b_1(3Z^2 - 2Z) + b_2(Z - a_1)^2 I_1 + b_3(Z - a_2)^2 I_2 \quad (1)$$

$$\left(\frac{d}{D}\right)^2 = Z^2(b_1 + b_2 \cdot Z) + (Z - a)^2\left[b_3 + b_4(Z - 2a)I\right] \quad (2)$$

$$\left[\frac{d}{D}\right]^2 = b_1(T\text{-}1) + b_2\left[\frac{h^2}{H^2}\text{-}1\right] + b_3\left[a_1\text{-}T\right]I_1 + b_4\left[a_2\text{-}T\right]I_2 \quad (3)$$

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[1] The symbols used in the taper functions besides using diameter at breast height, *d*, and total height, *h*, are the following:

h -height on the stem(m); d -diameter at stem height h_i (cm)

$$z = \left(\frac{H-h}{H}\right)$$

$$k = (3.141592/40000)$$

$$V_m = aD^{b1}H^{b2}$$

$$y = 2z[3\cdot(z^2) - 2z] + [4\cdot(z^3) - 2z] + [5\cdot(z^4) - 2z] + [6\cdot(z^5) - 2z]$$

$$x = \left[\frac{H-h}{H-1.3}\right]$$

$$T = \left[\frac{h}{H}\right]$$

$$I = 1\,if\,\frac{h}{H} \le a\;and\;I = 0\,otherwise$$

$$I_1 = 1\,if\,\frac{h}{H} \le a_1\;and\;I_1 = 0\,otherwise$$

$$I_2 = 1\,if\,\frac{h}{H} \le a_2\;and\;I_2 = 0\,otherwise$$

$$\left(\frac{d}{D}\right)^2 = b_1(T-1) + b_2(T^2-1) \tag{4}$$

$$\left(\frac{d}{D}\right)^2 = b_0 + b_1T + b_2T^2 \tag{5}$$

$$\frac{d^2\cdot K\cdot H}{V_m} = \left[b_0\cdot 2Z + b_1(3Z^2-2Z) + b_2(4Z^3-2Z) + b_3(5Z^4-2Z) + b_4(6Z^5-2Z)\right] \tag{6}$$

$$d = b_1 D^{b_2}(H-h)^{b_3} H^{b_4} \tag{7}$$

$$\left(\frac{d}{D}\right)^2 = b_1\left(\frac{1}{D^2\cdot H}\right) z^{b_2} + b_3 z^{b_4} \tag{8}$$

$$\frac{d}{D} = b_1 x^{b_2} \tag{9}$$

$$\frac{d}{D} = x\exp\left[b_1 + b_2x + b_3\left(\frac{D}{H}\right)^2 + b_4x^2\frac{D}{H} + b_5\sqrt{T} + b_6DT\right] \tag{10}$$

$$d = b_1\left[\frac{D(H-h)}{H-1.3}\right] + b_2\left[\frac{(H^2-h^2)(H-1.3)}{H^2}\right] \tag{11}$$

$$d = D\left[b_1 + b_2\ln\left(1-\exp\left(\frac{-b_1}{b_2}\right)T^{1/3}\right)\right] \tag{12}$$

where b_i are statistical parameters to be estimated.

The statistics for sampled trees were depicted in Table 1. Minimum and maximum top heights were measured in *P. hartwegii* and *P. pseudostrobus* of Nuevo Leon, Mexico. Minimum diameters at breast height were measured in *P. ayacahuite* and *Quercus sp.* of Durango, Mexico.

Four quality statistics widely recommended (Newnham 1990) were used for the evaluation of the goodness of fit and for comparisons of alternative models: (i) the standard error of the estimate *Sx* (ii) mean absolute error *e*, (iii) the percent error *S(%)* and (iv) the coefficient of determination r^2. The goodness-of-fit is reported as follows:

$$Sx = \sqrt{\frac{\sum_{j=1}^{n}\sum_{i=1}^{n}\left(d_{ij}-\bar{d}_j\right)^2}{n-p}} \tag{13}$$

$$e = \left|d_{ij}-\hat{d}_{ij}\right| \tag{14}$$

$$S(\%) = \frac{100}{n}\sum_{j=1}^{n}\sum_{i=1}^{n}\left|d_{ij}-\hat{d}_{ij}\right| / \hat{d}_{ij} \tag{15}$$

$$r^2 = 1-\sum_{j=1}^{n}\frac{\sum_{i=1}^{n}\left(d_{ij}-\hat{d}_{ij}\right)^2}{\sum_{i=1}^{n}\left(d_{ij}-\bar{d}_j\right)^2} \tag{16}$$

where: d_{ij} - measured diameter at h_i of tree j (cm), $\hat{d}_{ij}$ - estimated diameter at h_i of tree j

(cm), $\bar{d}_j$ - mean diameter of tree j (cm), n - sample size.

In order to facilitate computations, a single taper model is selected based on the consistency of quality statistics. Hence, comparisons between taper functions were conducted to evaluate the statistical efficiency in estimating diameter i. Taper models with the least mean errors (Sx, e, $S(\%)$) and the largest mean coefficients of determination for all species were chosen to evaluate merchantable volume components of 62 forest stands of the Sierra Madre Occidental mountain range.

Once the single equation was selected, the confidence intervals were calculated on estimated parameters with the standard errors. Confidence intervals used an error of 0.05 with degrees of freedom equal the number of observations minus p -2, where p - number of parameters to be estimated. Since these bounds were quite small, the standard error was plotted for each parameter, in order to observe similitudes and differences between parameters of different tree species.

Merchantable volume components were calculated using the selected stem profile function, using average parameters for pines of Durango and Chihuahua and for oaks. This task was conducted on the 62 stands inventoried in temperate forests of the Sierra Madre Occidental mountain range of Central Durango. The merchantable volume components estimated were: (i) poles and fences (trees with DBH < 20 cm), (ii) sawn wood (tress with 20 ≤ DBH ≤ 40 cm, and $d_i > 20$ cm), (iii) plywood (trees with DBH > 40 cm and $d_i > 40$ cm), and (iv) secondary forest products (tips of stems with d_i <20 cm).

When estimated the total tree volume was contrasted by the conventional volume equations of Schumacher & Hall (1933) and by the numerical integration of the selected taper equation. Please note that the Newnham (1990) taper equation cannot be analytically integrated. The numerical integration was conducted on stem lengths of 0.010 m. The stem volume equations are the following: (i) for pine trees: $V_s = 0.000054 \cdot D^{1.9693} \cdot H^{0.9589}$ and, (ii) for oak trees: $V_s = 0.000079 \cdot D^{1.9123} \cdot H^{0.8139}$.

Results

Using mean statistics, the taper model of Newnham (1990) given by equation (9) resulted in highest r^2 values and smallest errors (Tables 2-5). The models of Max and Burkhart (1976), Goulding & Murray (1976), Newnham (1988), Parresol (1987) and Clutter (1980) reported also excellent mean quality statistics but unfortunately they did performed well only for specific tree species.

Statistical parameters and their standard errors for Newnham (1990) taper model of each tree species are reported in Table 6.

The b_1 and b_2 values vary among the species, as was observed early by Baskerville (1965) and later by Zianis & Mencuccini (2004) for other allometric equations. Most notorious difference was on slope (b_2) between oaks and pines, since mean values approached 0.92 and 0.63, respectively (Figure 1). For pines, coefficients b_1 and b_2 are related by a linear equation: $b_1 = 2.24 + 273b_2$; $r^2 = 0.53$, in agreement with relationships between coefficients of other allometric equations (Zianis & Mencuccini 2004, Pilli et al. 2006, Návar 2009).

The confidence bounds showed each single tree species has its own stem profile (p = 0.0001). Oak species *P. leiophylla*, *P. ayacahuite*, and other species do have stems portraying better a cone than a cylinder, since has values e.g. $b_2 > 0.70$ unlike *P. durangensis*, *P. cooperi, P. arizonica, P. teocote, P. pseudostrobus*, *P. hartwegi*i with $b_2 < 0.60$ whose timber resembles more a cylinder than a cone. *P. ayacahuite*, *P. leiophylla* and other spp (*Pseudotsuga menziesii*) are secondary species in these forests and trees probably grow at a faster peace in height in order to reach full sunlight. Hence, they have an acute tapering. *P. cooperi*, *P. arizonica*, *P. hartwegii*, *P. durangensis* and

Table 2 The mean standard errors for 12 taper functions fitted to stem profiles of tree species of temperate forests of Northern Mexico

Taper Model	Ospp	Pariz	Paya	Pdur	Phar	Plei	Ppse	Pteo	Qspp	Pcoo
Cao et al. (1980)	0.96	1.44	0.96	2.44	1.03	993	9.24	31.50	2.28	697
Parresol & Thomas (1996)	1.10	2.48	1.10	2.71	1.03	5.25	3.29	2.58	2.69	6.85
Max & Burkhart (1976)	1.21	1.58	1.21	2.12	1.03	3.72	2.50	1.91	2.55	3.79
Kozak (1969)	1.62	2.10	1.62	2.52	1.03	3.95	2.66	2.15	3.03	3.93
Kozak (1969)	1.90	2.51	1.90	2.96	1.03	3.63	3.00	2.38	3.50	5.21
Goulding & Murray (1976)	1.27	1.68	1.27	2.16	1.03	4.87	2.27	1.74	2.39	3.85
Clutter (1980)	1.43	2.05	1.43	2.45	1.03	2.94	2.55	2.08	2.79	3.72
Damaerschalk (1972)	1.69	2.09	1.69	2.65	1.03	3.92	2.59	2.12	3.25	3.81
Newnham (1990)	1.47	2.05	1.47	2.45	1.03	3.32	2.58	2.09	2.82	3.73
Newnham (1988)	1.20	1.81	1.20	2.10	1.03	3.76	2.52	1.92	2.75	67.83
Amidon (1984)	1.63	3.50	1.63	3.85	1.05	3.88	4.37	3.52	2.80	8.10
Biging (1984)	2.28	3.80	2.28	3.85	1.06	5.45	4.36	3.83	3.75	5.98

Note. Abbreviations: Ospp - other coniferous species, Pariz - *Pinus arizonica*, Paya - *P. ayacahuite*, Pdur - *P. durangensis*, Phar - *P. hartwegii*, Plei - *P. leiophylla*, Ppse - *P. pseudostrobus*, Pteo - *P. teocote*, Qspp - *Quercus sp.*, Pcoo - *P. cooperi*.

Table 3 The mean absolute errors for 12 taper functions fitted to stem profiles of tree species of temperate forests of Northern Mexico

Taper Model	Ospp	Pariz	Paya	Pdur	Phar	Plei	Ppse	Pteo	Qspp	Pcoo
Cao et al. (1980)	9.12	6.58	9.12	10.04	9.07	67.67	11.75	51.79	10.99	68.67
Parresol & Thomas (1996)	17.63	19.44	17.63	24.58	18.17	49.40	20.82	22.70	18.56	49.33
Max & Burkhart (1976)	9.92	7.19	9.92	8.95	10.09	15.12	9.98	10.88	12.23	11.80
Kozak (1969)	11.87	9.26	11.87	11.46	10.59	17.23	11.24	12.19	26.67	12.56
Kozak (1969)	19.06	15.50	19.06	16.91	10.75	18.35	16.50	17.48	21.24	17.79
Goulding & Murray (1976)	10.23	7.65	10.23	8.79	8.72	13.71	9.13	10.33	11.31	9.63
Clutter (1980)	11.55	8.74	11.55	10.84	11.05	14.90	10.86	11.88	17.83	11.07
Damaerschalk (1972)	13.10	9.20	13.10	12.38	10.44	21.03	11.01	12.51	13.98	11.17
Newnham (1990)	12.55	8.62	12.55	11.08	10.53	16.97	11.17	12.14	17.71	11.16
Newnham (1988)	13.75	9.05	13.75	13.29	12.48	17.48	11.92	14.38	16.34	11.20
Amidon (1984)	23.20	32.43	23.20	35.86	27.08	26.37	30.61	32.17	20.67	31.70
Biging (1984)	23.35	21.37	23.35	21.68	24.93	27.22	21.31	23.06	23.44	135

Note. Abbreviations: Ospp - other coniferous species, Pariz - *Pinus arizonica*, Paya - *P. ayacahuite*, Pdur - *P. durangensis*, Phar - *P. hartwegii*, Plei - *P. leiophylla*, Ppse - *P. pseudostrobus*, Pteo - *P. teocote*, Qspp - *Quercus spp*, Pcoo - *P. cooperi*.

P. pseudostrobus are pioneer tree species and they quickly reach a dominant position in stand and hence presents a cylinder-type stem profile in contrast to the rest of the species (Figures 1-2).

Diameters of *Quercus* species appear to decay following a straight line or a parabolic equation (Table 6, Figure 2). The stem profiles of *P. hartwegii* are similar to those of *Quercus* species, although the former presents a strong tapering at the tip (1-1.5 m of the tip) than the later. *P. hartwegii* is found only in the upper most sites of the Sierra Madre Oriental mountain range of Nuevo Leon and Coahuila, above 3000 m above sea level. Therefore they are shorter, with larger basal diameters, and with an acute tapering at the tip than the rest of the pine species.

Table 4 The mean percent errors for 12 taper functions fitted to stem profiles of tree species of temperate forests of Northern Mexico

Taper Model	Ospp	Pariz	Paya	Pdur	Phar	Plei	Ppse	Pteo	Qspp	Pcoo
Cao et al. (1980)	39.86	43.89	39.86	639.00	32.78	20715.00	1676.00	724.00	71.49	9439.00
Parresol & Thomas (1996)	42.30	43.62	42.30	508.00	35.44	86.83	45.59	42.00	71.79	76.36
Max & Burkhart (1976)	57.16	63.19	57.16	641.00	54.52	163.00	67.31	92.68	98.06	95.20
Kozak (1969)	345.00	438.00	345.00	518.00	190.00	692.00	539.00	451.00	605.00	786.00
Kozak (1969)	54.46	51.05	54.46	83.71	45.73	78.60	53.44	64.27	65.85	84.87
Goulding & Murray (1976)	64.57	61.63	64.57	645.00	38.56	138.00	63.44	54.63	100.00	106.00
Clutter (1980)	43.62	49.93	43.62	612.00	38.31	144.00	59.07	74.76	68.88	111.00
Damaerschalk (1972)	51.10	48.51	51.10	564.00	39.73	156.00	55.42	69.65	122.00	119.00
Newnham (1990)	45.21	51.71	45.21	599.00	40.57	120.00	54.83	69.87	69.31	109.00
Newnham (1988)	44.19	49.38	44.19	546.00	36.81	120.00	69.93	50.72	78.37	1065.00
Amidon (1984)	45.64	51.97	45.64	456.00	43.24	115.00	55.43	49.28	65.75	87.64
Biging (1984)	87.08	112.00	87.08	687.00	101.00	218.00	117.00	172.00	123.00	206.00

Note. Abbreviations: Ospp - other coniferous species, Pariz - *Pinus arizonica*, Paya - *P. ayacahuite*, Pdur - *P. durangensis*, Phar - *P. hartwegii*, Plei - *P. leiophylla*, Ppse - *P. pseudostrobus*, Pteo - *P. teocote*, Qspp - *Quercus spp*, Pcoo - *P. cooperi*

Table 5 The mean coefficient of determinations for 12 taper functions fitted to stem profiles of tree species of temperate forests of Northern Mexico

Taper Model	Ospp	Pariz	Paya	Pdur	Phar	Plei	Ppse	Pteo	Qspp	Pcoo
Cao et al. (1980)	0.98	0.98	0.98	0.96	0.97	0.84	0.45	0.93	0.97	0.90
Parresol & Thomas (1996)	0.98	0.94	0.98	0.95	0.95	0.77	0.93	0.93	0.96	0.72
Max & Burkhart (1976)	0.98	0.98	0.98	0.97	0.96	0.88	0.96	0.96	0.96	0.91
Kozak (1969)	0.96	0.96	0.96	0.95	0.95	0.87	0.95	0.95	0.94	0.91
Kozak (1969)	0.94	0.94	0.94	0.94	0.95	0.89	0.94	0.94	0.93	0.84
Goulding & Murray (1976)	0.97	0.97	0.97	0.97	0.97	0.80	0.97	0.97	0.97	0.91
Clutter (1980)	0.97	0.96	0.97	0.96	0.95	0.93	0.96	0.95	0.95	0.92
Damaerschalk (1972)	0.95	0.96	0.95	0.95	0.96	0.87	0.96	0.95	0.94	0.91
Newnham (1990)	0.96	0.96	0.96	0.96	0.95	0.91	0.96	0.95	0.95	0.92
Newnham (1988)	0.98	0.97	0.98	0.97	0.95	0.88	0.96	0.96	0.95	0.96
Amidon (1984)	0.96	0.89	0.96	0.89	0.89	0.87	0.88	0.87	0.95	0.61
Biging (1984)	0.92	0.87	0.92	0.89	0.81	0.75	0.88	0.84	0.92	0.79

Note. Abbreviations: Ospp - other coniferous species, Pariz - *Pinus arizonica*, Paya - *P. ayacahuite*, Pdur - *P. durangensis*, Phar - *P. hartwegii*, Plei - *P. leiophylla*, Ppse - *P. pseudostrobus*, Pteo - *P. teocote*, Qspp - *Quercus spp*, Pcoo - *P. cooperi*

Visual analysis of Figure 2 shows that for small trees ($D \leq 20$ cm), the stem profiles of *P. ayacahuite*, *P. durangensis*, *P. pseudostrobus*, and *P. teocote* are quite similar, but for larger trees ($D \leq 40$ cm) only *P. pseudostrobus* and *P. teocote* trees remain with similar tapering. *P. ayacahuite* presents larger tapering for these trees and appears to be explained by the D vs H relationship. Thus, *P. ayacahuite* trees attain largest top heights than any other pine species for similar diameter at breast height. This species can be found mostly in humid sites, growing most of the time below the canopy of other pine trees and grows quickly in height to attain a dominant sociological position in the stand.

Summary statistics for merchantable volume components for 62 forest stands are reported in Table 7. Poles and fences, sawn wood, plywood and secondary forest products accounts for 14, 46, 26, and 13% of the total stem volume, respectively. Sawn wood and plywood account for by 73% of the total stem stand volume.

Total stem volume/ha estimates for 62 forest stands were statistically similar, since mean (confidence interval) timber volume was 347 (36) m^3 ha^{-1} and 344 (36) m^3 ha^{-1} when using the Schumacher and Hall (1933) volume equations and when numerically integrating the Newnham (1990) taper function respectively. In fact, the difference between both timber volume/ha estimates is distributed normally, according to the Shapiro Wilks (p = 0.4495) and the Kolmogorov-Smirnoff (p = 0.15) statistics. Mean difference was 3.13 m^3 ha^{-1} and it is less than 1% of the mean stand timber estimated by any procedure. Since Martin (1984) and Figuereido and Burkhart (1999) noted that measured timber volume by immersion procedures is statistically similar to timber volume

Table 6 Coefficients of the Newnham (1990) taper model and their standard errors for tree species of northern Mexico

Species	Parameters		
	Statistic	b_1	b_2
Quercus spp	Mean	0.010400	0.916300
	S.E.	0.000036	0.011100
Pinus teocote	Mean	0.010300	0.595400
	S.E.	0.000017	0.003200
Pinus pseudostrobus	Mean	0.010300	0.578900
	S.E.	0.000024	0.004450
Pinus leiophylla	Mean	0.011100	0.752400
	S.E.	0.000062	0.013500
Pinus hartwegii	Mean	0.010300	0.581200
	S.E.	0.000029	0.005490
Pinus durangensis	Mean	0.010400	0.581500
	S.E.	0.000018	0.003370
Pinus ayacahuite	Mean	0.010600	0.749800
	S.E.	0.000052	0.012800
Pinus arizonica	Mean	0.010500	0.558800
	S.E.	0.000024	0.004760
O. species	Mean	0.010600	0.749800
	S.E.	0.000052	0.012800
Pinus cooperi	Mean	0.010400	0.503100
	S.E.	0.000054	0.009100

Note. Abbreviation: S.E. - standard error of mean.

Table 7 Summary statistics of merchantable volume for different utilizations from the 62 stands of temperate, mixed, uneven-aged forests of Central Durango, Mexico

	Merchantable volume components (m^3 ha^{-1})				
	Total	Poles & fences	Sawn wood	Plywood	Secodary forest products
Mean	343.70	48.510	159.14	90.09	45.99
Standard Deviation	143.50	29.150	83.91	81.57	23.24
Confidence Interval (α = 0.05)	35.72	7.255	20.89	20.30	5.78

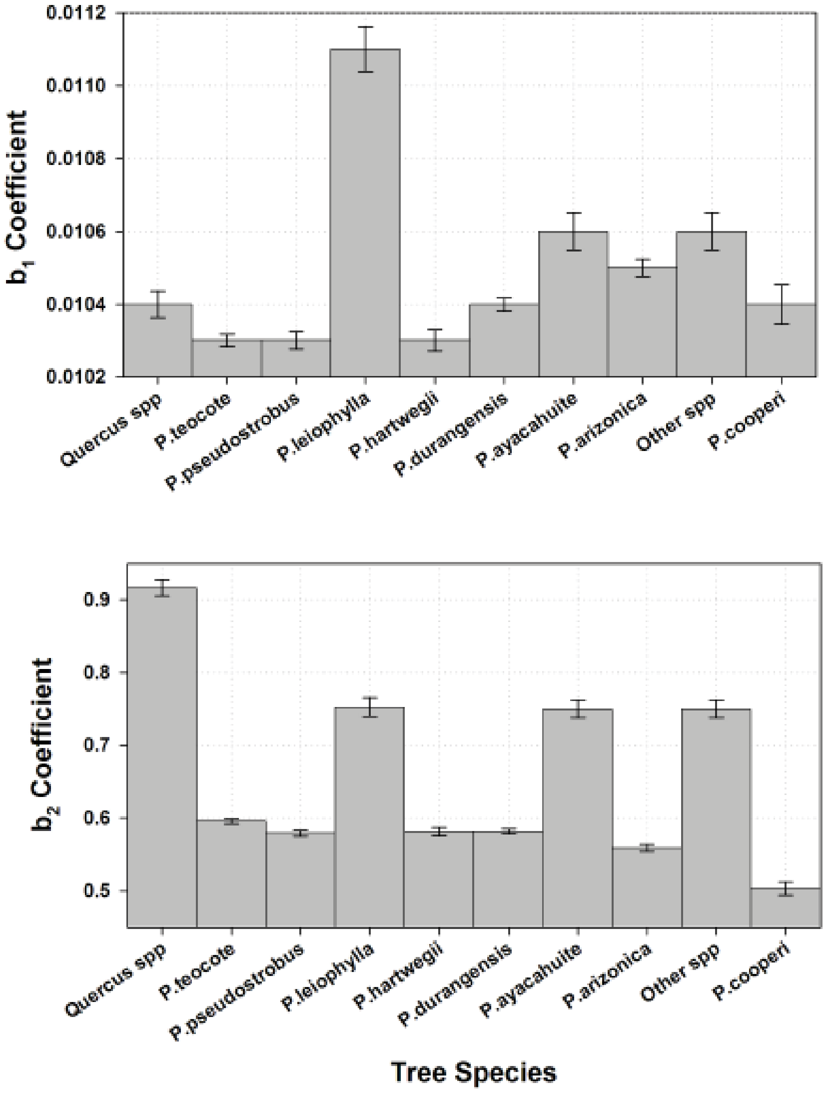

Figure 1 Parameter estimates and standard errors for 10 tree species of northern Mexico (see Table 6)

estimates by volume equations, then it can be concluded that the taper equation provides sound total timber volume estimates.

Discussion

The large sample size, the ample regional distribution of the species harvested, coupled with a large variation in diameter and top height values provide accurate estimates of taper equations and merchantable volume as well as provides insight into the dynamics of tree allometry that would not have been forthcoming had we sampled fewer numbers of larger trees. According to the confidence intervals on the Newnham (1990) equation parameters, species-specific equations were derived for temperate tree species of northern Mexico.

The taper function of Newnham (1990) is satisfactory and consistent predictor of stem profiles of all studied species since mean statistics were lower for the errors and larger for the coefficient of determination for all species. The total variation explained by the model was consistently above 91%, with an average (± confidence interval) of 95% (± 0.01%). The statistical stem profile relationship between

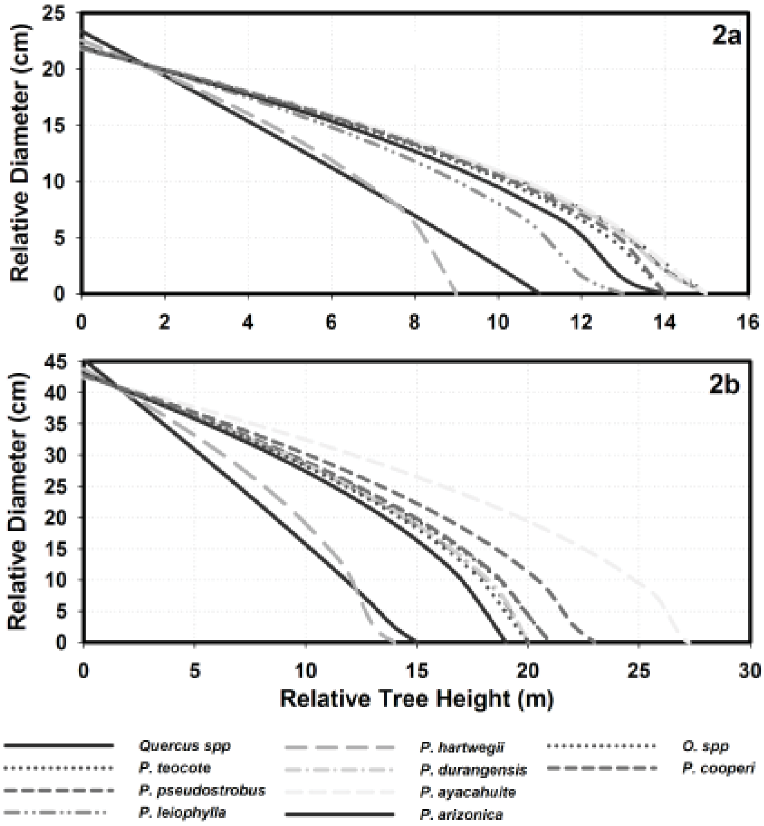

Figure 2 Taper function of Newnham (1990) applied to trees of 20 (2a) and 40 cm (2b) of diameter at breast height for 10 tree species of northern Mexico

h and *d* is better predicted by the Newnham (1990) equation for *P. arizonica*, *P. ayacahuite*, *P. durangensis* and *P. pseudostrobus*; those trees with the largest variation in *D* and the largest sample size (Table 1). *P. cooperi* and *P. leiophylla* had the smallest variance accounted for by the taper models and appears to be related to smaller sample size, as well as to the wide dispersion of trees harvested across the Sierra Madre Occidental mountain range, from southern Durango (Ejido Teneraca in Mezquital, Durango, Mexico) to southern Chihuahua, Mexico (Ejido Guachochi, Chihuahua, Mexico).

However, when considering all individual statistics for each tree species, different equations predicted better the stem profile of single tree species. Using four quality statistics, the segmented equation of Cao et al. (1980) is a better predictor of the stem profiles for *P. arizonica*, *P. ayacahuite, Quercus* sp., and other species. The polynomial equation of Goulding and Murray (1976) and the variable exponent function of Newnham (1988) describe better the stem profiles for *P. durangensis* trees. The equation of Goulding and Murray (1976) predicted better the stem profiles of *P. hartwegii*, *P. pseudostrobus*, *P. teocote*, *P. cooperi*, and *P. leiophylla* trees.

Despite all the advances in the mathematical description of the stem profiles, it is currently difficult to identify a single taper function or family of taper functions which present a clear advantage over the rest. The model of Bigging (1984) proved to be better predictor of the stem profiles of *P. cooperi*, *P. engelmannii*, *P. leiophylla*, *P. herrerae*, and *P. durangensis* of temperate forests of Central Durango, Mexico (Corral-Rivas et al. 1999). These authors noted that several taper functions described better stem profiles of specific pine species, although they chosed a segmented model that consistently fitted empirical data better for pines of temperate forests of central Durango, Mexico. Therefore, new taper models are continuously being developed to try to explain the intrinsic and extrinsic variation.

Competition plays a key role in modulating tapering of trees. Contrary to the visual observations in the field, Calama and Montero (2006) noted that trees growing free from competition were less tapered than trees growing under large stocking densities. The rationale behind this statistical finding was that stand density or social position affect stem form through crown ratio and length, and that stem within crown tends to be more tapered than branch-free bole, due to the contribution of branches to stem growth. On the other hand, in highly stocked stone pine stands, the crown pattern and branching development tend to be similar to other coniferous species (Mutke *et al.* 2005), maintaining a single apical dominance which leads to larger crown ratios.

Regardless of the goodness of fit, almost all taper models have some sort of unexplained variation. The level of non-explained random variability in stem diameter indicates the presence of factors that have not been considered and that partially controls stem form, which act at plot level (e.g. ecological factors or silvicultural treatments) or tree level (e.g. microsite, distance dependent competition, crown habitat and size, or genetics).

The pattern of variability of stem form between trees located in different plots has been explained by the inclusion of stand variables in the stem profile equation (Lappi 1990, Muhairwe et al. 1994). The sociological position of the tree within the same plot also explains some sort of variability (Kozak 1988, Newnham 1992, Valentine & Gregoire 2001). Unfortunately, none of these variables at the stand scale or sociological position of trees harvested for measurements were measured during the course of this research.

Tapering, and therefore the distribution of forest products, is variable between trees of the same stand, between trees of different stands and between species. In this study, a tree of 40 cm *dbh* has approximately 29 (41%); 66 (52%); and 5 (7%) of the total volume in plywood, sawn wood and secondary forest products, respectively if it is a pine (or an oak tree) species.

The Newnham (1990) stem profile equations provide mean estimates of 49, 159, 90 and 46 m^3 ha^{-1} for poles, sawn wood, plywood and secondary forest products. The average estimate for total timber volume at the stand level is 14%, 46%, 23%, and 13%. Mean timber volume data for 637 circular 1/10 ha plots was 135 m^3 ha^{-1}, of which 129 m^3 ha^{-1} are in the timber harvested and 6 m^3 ha^{-1} are in stumps left in the forest floor. The structure of the timber available for harvesting in temperate forests of Central Durango, Mexico based on estimated percentages was 18 m^3 ha^{-1}, 59 m^3 ha^{-1}, 30 m^3 ha^{-1} and 17 m^3 ha^{-1} for poles, sawn wood, plywood and secondary forest products, respectively. This information is useful for forest industry that aims at optimization of forest products derived from timber harvest as well as for estimating the life time of end forest products.

Conclusions

In this research, 12 taper models fitted stem profile data for 1640 trees from 10 Mexico's northern temperate tree species. The power taper equation of Newnham (1990) fitted well the diameter-height relationship, since the total stand volume deviated by less than 1%, from total stem volume estimated from conventional stem volume, for 62 forest stands inventoried in Central Durango, Mexico. Using the Newnham (1990) taper model from the mean timber volume (129 m^3 ha^{-1}) inventoried in temperate forests of Central Durango (637 plots), 18 m^3 ha^{-1}, 59 m^3 ha^{-1}, 30 m^3 ha^{-1}, and 17 m^3 ha^{-1} are estimated to be poles, sawn wood, plywood and secondary forest products, respectively.

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